Çankaya University – ECE Department – ECE 376

Student Name : Open source exam
Student Number : Duration : 2 hours

Questions

1. (35 Points) An 8 PSK signal of 20 M symbols / sec is given. This signal is used as an input to an OFDM system, where the transmitter output passes through a communication channel with (nearly) flat frequency responses of 3 MHz. Determine the minimum number of subcarriers to be used and the frequencies of these subcarriers. By assuming that the symbol $\mathbf{s}_4$ of the 8 PSK is placed on the first subcarrier, show how a successful demodulation can be performed at the receiver.

Solution : Assuming the inverse of the bit/symbol duration is approximately equal to the required bandwidth, then for a $R = 20$ M symbols / sec, Mary signal, we would require a bandwidth of

$$B_c = R = 1/T_s = 20 \text{ MHz} \quad , \quad T_s = 50 \text{ nsec} \tag{1.1}$$

It is given that such a bandwidth is available, but with slices of flat response of 3 MHz. Then we need to generate

$$K = 20 \text{ MHz} / 3 \text{ MHz} \approx 7 \text{ OFDM subcarriers} \tag{1.2}$$

In line with our previous solution and related lecture notes, we increase $K = M = 8$, hence the duration of the OFDM symbols (after conversion from 8 PSK serial stream to parallel data), we have

$$T = MT_s = 0.4\ \mu\text{sec} \tag{1.3}$$

For orthogonality to be satisfied whilst remaining within the 20 MHz given bandwidth, our subcarriers must have an integer number of cycles within $T == 0.4\ \mu\text{sec}$. Hence a simple listing is

$$\begin{aligned} &f_1 = \frac{1}{T} = 2.5 \text{ MHz}\ ,\ f_2 = \frac{2}{T} = 5 \text{ MHz}\ ,\ f_3 = \frac{3}{T} = 7.5 \text{ MHz}\ ,\ f_4 = \frac{4}{T} = 10 \text{ MHz} \\ &f_5 = \frac{5}{T} = 12.5 \text{ MHz}\ ,\ f_6 = \frac{6}{T} = 15 \text{ MHz}\ ,\ f_7 = \frac{7}{T} = 17.5 \text{ MHz}\ ,\ f_8 = \frac{8}{T} = 20 \text{ MHz} \end{aligned} \tag{1.4}$$

For the frequencies listed in (1.4), it is easy to verify that

$$f_p - f_m = \frac{p-m}{T} \quad , \quad 1 \le m < M \quad , \quad p = m + i \quad , \quad 1 \le i < M - m \tag{1.5}$$

As done in the following m file named Q1ort_FE_31052017.m

```
clear;clc ;clf reset;close all
syms t
T = 0.4;fsub = 2.5:2.5:20;M = 8;Iarr = [];Iearr = [];
for m = 1:M-1
    for i = 1:M - m
        p = m + i;
c1 = cos(2*pi*fsub(p)*t);c2 = cos(2*pi*fsub(m)*t);
c1e = exp(j*2*pi*fsub(p)*t);c2e = exp(-j*2*pi*fsub(m)*t);
I = int(c1*c2,t,0,T);Iarr = [Iarr I];
I = int(c1e*c2e,t,0,T);Iearr = [Iearr I];
end;end

if Iarr == 0;disp('OK');end
if Iearr == 0;disp('OK');end
```

Note that in the frequency listing of (1.3), both the given channel bandwidth is not exceeded, additionally adjacent frequency spacing is less than the 1 MHz slices of the total channel bandwidth, namely

$$f_8 - f_1 = 17.5 \text{ MHz} < B_c = 20 \text{ MHz} \quad , \quad f_p - f_{p-1} = 2.5 \text{ Hz} < 3 \text{ MHz} \tag{1.6}$$

The symbols of 8 PSK placed on a unit circle are given below in complex notation

$$\mathbf{s}_1 = 1,\ \mathbf{s}_2 = \frac{1+j}{\sqrt{2}},\ \mathbf{s}_3 = j,\ \mathbf{s}_4 = \frac{-1+j}{\sqrt{2}}, \mathbf{s}_5 = -1,\ \mathbf{s}_6 = \frac{-1-j}{\sqrt{2}},\ \mathbf{s}_7 = -j,\ \mathbf{s}_8 = \frac{1-j}{\sqrt{2}} \tag{1.7}$$

Upon multiplying the subcarriers in (1.4) by 0.4×10^{-6}, we have

$$\begin{aligned} &f_{1d} = 1 \text{ Hz},\ f_{2d} = 2 \text{ Hz},\ f_{3d} = 3 \text{ Hz},\ f_{4d} = 4 \text{ Hz} \\ &f_{5d} = 5 \text{ Hz},\ f_{6d} = 6 \text{ Hz},\ f_{7d} = 7 \text{ Hz},\ f_{8d} = 8 \text{ Hz} \end{aligned} \tag{1.8}$$

Taking one OFDM symbol interval, we have on the transmitter side, after modulation

$$\begin{aligned} &y(n) = \frac{1}{N}\sum_{k=1}^{K} {}_k\mathbf{s}_m \exp\left(j2\pi f_{kd} n / N\right) = \frac{1}{N}\sum_{k=1}^{8} {}_k\mathbf{s}_m \exp\left(j2\pi f_{kd} n / N\right) \\ &N = 8,\ m = 1\cdots 8,\ n = 0\cdots 7 \end{aligned} \tag{1.9}$$

We choose the following arbitrary assignment of PSK symbols for 8 PSK

$$\begin{aligned} &{}_1\mathbf{s}_m = \mathbf{s}_4,\ {}_2\mathbf{s}_m = \mathbf{s}_4,\ {}_3\mathbf{s}_m = \mathbf{s}_2,\ {}_4\mathbf{s}_m = \mathbf{s}_1 \\ &{}_5\mathbf{s}_m = \mathbf{s}_1,\ {}_6\mathbf{s}_m = \mathbf{s}_7,\ {}_7\mathbf{s}_m = \mathbf{s}_8,\ {}_8\mathbf{s}_m = \mathbf{s}_5 \end{aligned} \tag{1.10}$$

Then for demodulation of the first symbols on the receiver side

$$d_1 = \sum_{n=0}^{N-1} y(n) \exp\left(-j2\pi f_{1d} n / N\right) = \mathbf{s}_4 + 0 + 0 + 0 + 0 + 0 + 0 + 0 = \frac{1+j}{\sqrt{2}} \tag{1.11}$$

The calculations are performed in bottom half of the Matlab file, Q1ort_FE_31052017.m.

2. (35 Points) Four spreading PN sequences are given below

$$\begin{aligned} \mathbf{e}_1 &= [1\,1\,-1\,1\,-1\,-1\,1] \quad , \quad \mathbf{e}_2 = [1\,1\,-1\,-1\,1\,-1\,1] \\ \mathbf{e}_3 &= [1\,1\,-1\,1\,1\,-1\,1] \quad , \quad \mathbf{e}_4 = [1\,1\,-1\,-1\,1\,1\,-1] \end{aligned} \tag{2.1}$$

The above PN sequences are used to spread four message signals. If $T_b = 1$ msec for these message signals, indicate the approximate bandwidth requirements before and after spreading. Evaluate and plot the normalized cyclic auto correlations of $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ and $\mathbf{e}_4$. Comment on which ones are maximum length.

Solution : From the PN sequences given in (2.1), we understand that $L_c = 7$, then $T_c = T_b / L_c = 1/7$ msec. From Fig. 1.3 of lecture notes entitled, Spread spectrum systems_2013_HTE", we see that

$$\begin{aligned} &\text{Before spreading : BW } \approx 2B_b \simeq 2/T_b = 2 \text{ kHz} \\ &\text{After spreading : BW } \approx 2B_c + 2B_b \simeq 2/T_b + 2/T_c = 16 \text{ kHz} \end{aligned} \tag{2.2}$$

The cyclic autocorrelation graphs of $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ and $\mathbf{e}_4$ are displayed in Figs. 2.1, 2.2, 2.3 and 2.4. Accordingly $\mathbf{e}_1$ and $\mathbf{e}_2$ are maximum length sequences, whereas $\mathbf{e}_3$ and $\mathbf{e}_4$ are not. The same can be detected by inspecting the PN sequences as follows

$$\begin{aligned} &\mathbf{e}_1 = \overbrace{[1\,1\,-1\,1\,-1\,-1\,1]}^{\text{no repeating subsequences}} \quad , \quad \mathbf{e}_2 = \overbrace{[1\,1\,-1\,-1\,1\,-1\,1]}^{\text{no repeating subsequences}} \\ &\mathbf{e}_2 = \left[\underset{T_c = 1/7 \text{ msec}}{\overbrace{1\;1\;-1}^{\text{same subsequence}}} \quad \overbrace{1\,1\,-1}^{\text{same subsequence}} \quad 1 \right] \quad , \quad \mathbf{e}_4 = \left[\underbrace{\overbrace{1\,1\,-1}^{\text{same subsequence}} \quad -1 \quad \overbrace{1\,1\,-1}^{\text{same subsequence}}}_{T_b = 1 \text{ msec}} \right] \end{aligned} \tag{2.3}$$

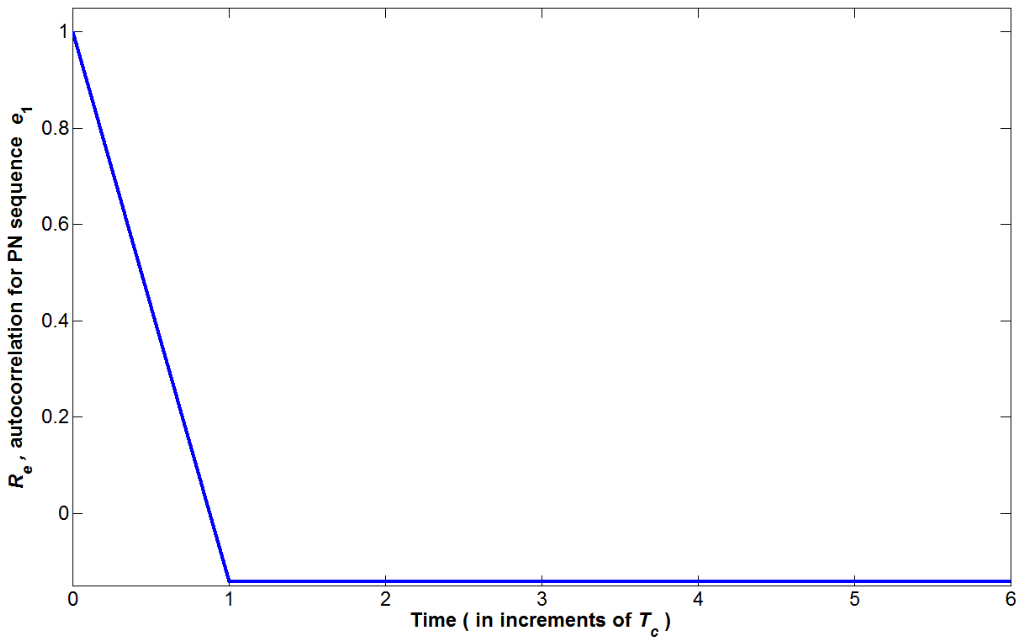

Fig. 2.1 Cyclic auto correlation of $\mathbf{e}_1$.

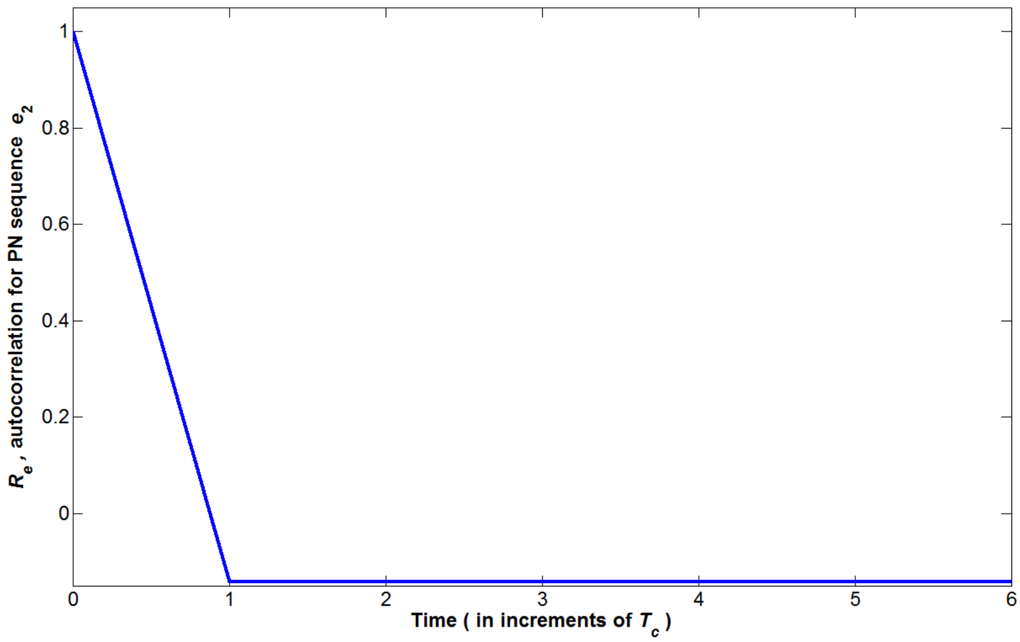

Fig. 2.2 Cyclic autocorrelation of $\mathbf{e}_2$.

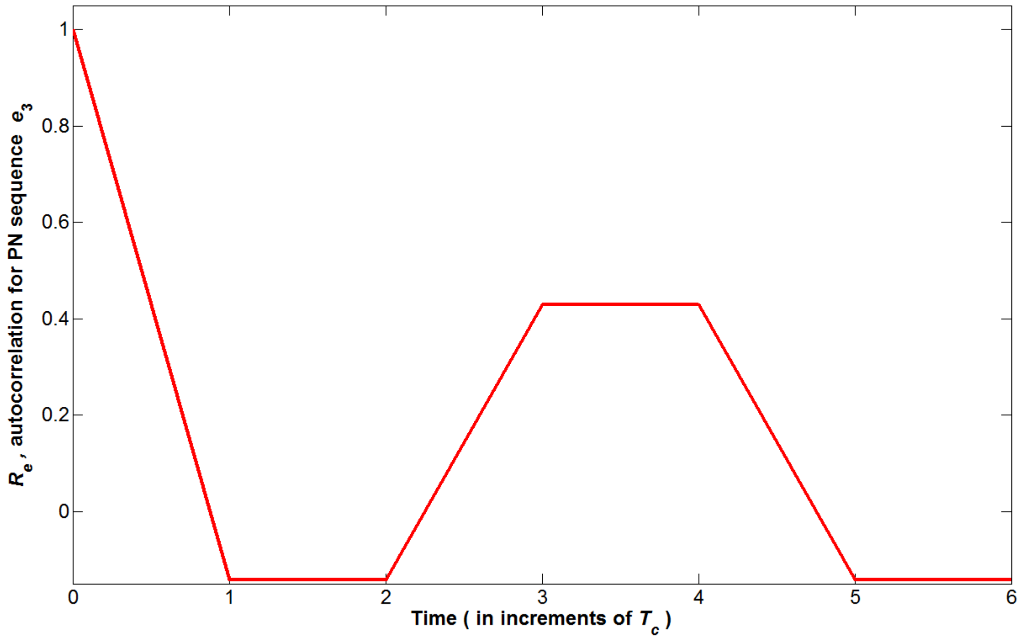

Fig. 2.3 Cyclic autocorrelation of $\mathbf{e}_3$.

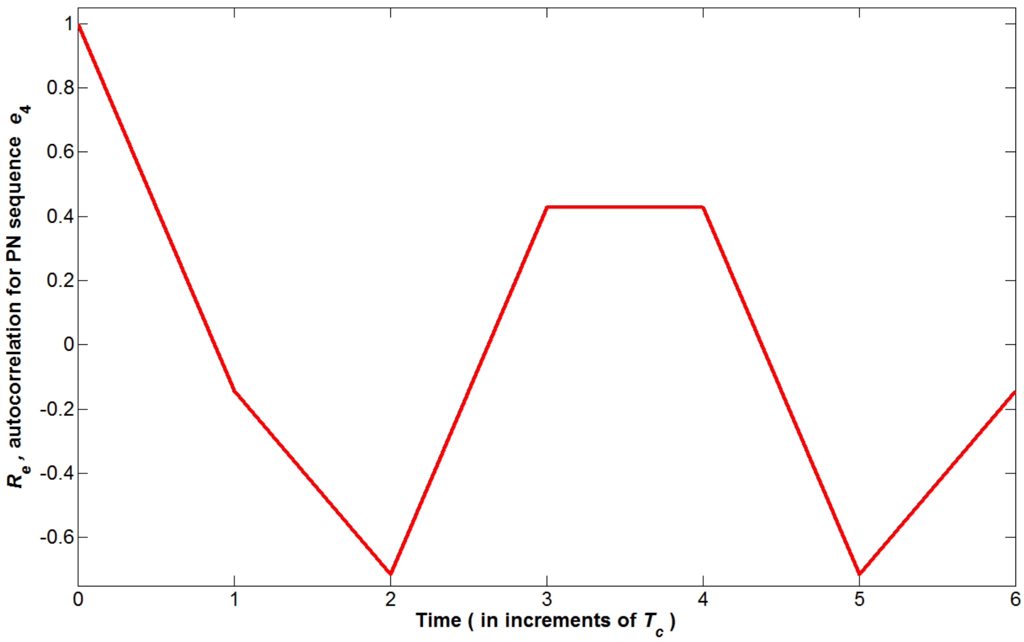

Fig. 2.4 Cyclic autocorrelation of $\mathbf{e}_4$.

The above is evaluated and plotted in the m file named, “Q2_FE_31052017.m”, whose code is reproduced below

```
clear;clc;close all;
e1 = [1 1 -1 1 -1 -1 1];
e2 = [1 1 -1 -1 1 -1 1];
e3 = [1 1 -1 1 1 -1 1];Lc = 7;fonsiz = 18;
e4 = [1 1 -1 -1 1 1 -1];esel1 = e4;esel2 = e4;
for i = 1:numel(esel1)
    corrvec(i) = esel1 * [esel2(i:end) esel2(1:i-1)]';
end
plot(0:numel(esel1)-1,corrvec/Lc,'-r','LineWidth',3);
set(gcf,'Color',[1 1 1]);set(gca,'FontSize',fonsiz)
xlabel('Time ( in increments of \itT_c\rm\bf
)','FontSize',fonsiz,'FontWeight','bold');
ylabel('\it\bfR_e , \rm\bfautocorrelation for PN sequence
\it\bfe\rm\bf_4','FontSize',fonsiz,'FontWeight','bold')
axis ([0 6 min(corrvec/Lc)*1.05 max(corrvec/Lc)*1.05]);
```

3. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones justify your answer.

a) Pulse position modulation can be multidimensional : True as explained in section 4.2 of ECE376_ Dimensionality of Signals_ASK_PSK_QAM_FSK_Jan 2013_HTE.

b) In OFDM, we use subcarrier frequencies which are integer multiples of each other : True but incomplete, these subcarriers must also be integer multiples of the inverse of OFDM symbol duration as specified in (3.2) and (3.3) of Notes on OFDM_2013.

c) QAM is used in cable TV broadcasting : In general QAM is used in cable and atmospheric radio links, since it uses the two dimensional signal space more efficiently than PSK.

d) OFDM uses orthogonal subcarriers to improve the error performance : Not necessarily, in OFDM the use of orthogonal subcarriers originates from the requirement of successful demodulation. Orthogonality of subcarrier allows the elimination of the interference of the symbols on the subcarriers other than the intended one during the demodulation process as shown in end of section 3 and in section 4 of Notes on OFDM_2013.

e) FM has better noise performance because of the differentiation process used in demodulation : True as illustrated in section 3.1 of ECE 376_AM_FM Demodulation_Jan 2013_HTE.