Çankaya University – ECE Department – ECE 376 (MT)

Student Name : Date : 27.03.2017
Student Number : Open Source Exam

Questions

1. (70 Points) The constellation diagram of the signal set, $s_1(t)$ to $s_8(t)$ is given in Fig. 1.1.

   a. Identify the type of modulation and dimensionality in this signal set. Write mathematical expression for $s_1(t)$ to $s_8(t)$ and plot them. Find the corresponding basis functions, $\psi_1(t)\ \cdots\ \psi_N(t)$ and plot $\psi_1(t)\ \cdots\ \psi_N(t)$. Write for the signal vectors $\mathbf{s_1}$ to $\mathbf{s_8}$, and plot the corresponding constellation diagram. Find the distance between signal vector ends.
   b. Draw the demodulator as correlator and matched filter. Assuming that the signal $s_1(t)$ is sent from the transmitted, find the outputs of the correlator and matched filter.
   c. Find the probability of error and decision regions via the evaluations of correlation metrics $C(\mathbf{r}, \mathbf{s}_m)$ again assuming $s_1(t)$ was transmitted.

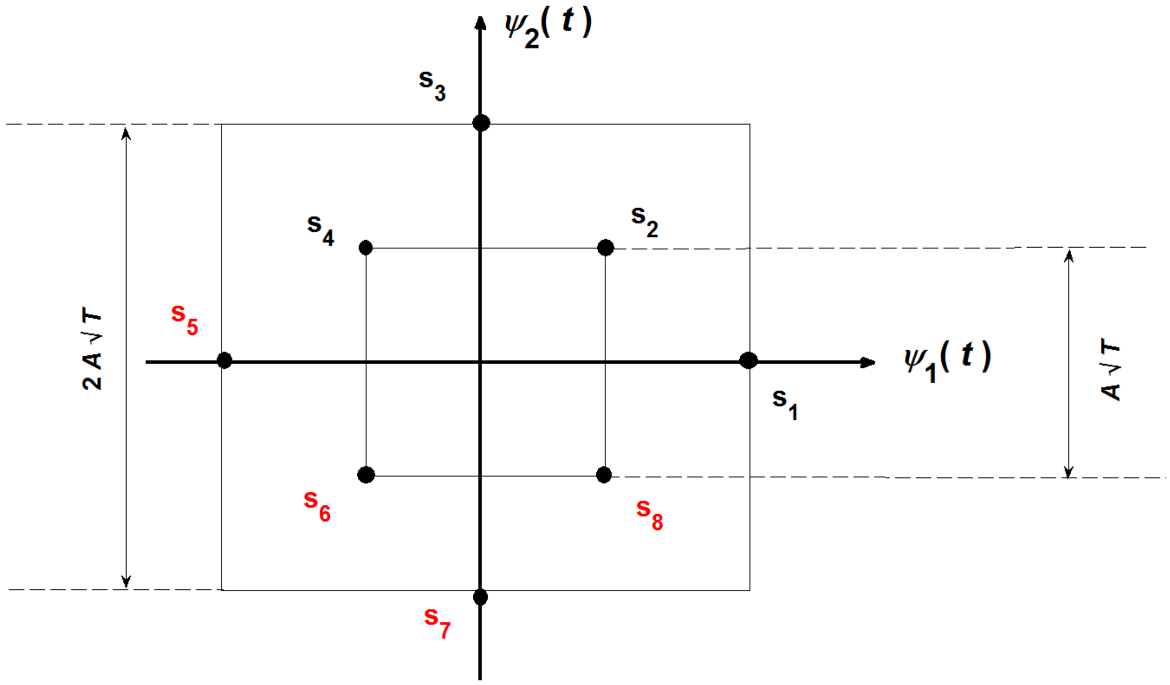

Fig. 1.1 The constellation diagram for Q1.

Solution : a. From the given constellation in Fig. 1, it is easy to see that we have 8 QAM The dimensionality is two.

So we can adapt the following common orthonormalized basis functions,

$$\psi_1(t) = \begin{cases} \sqrt{2/T} & 0 \le t \le T/2 \\ 0 & \text{otherwise} \end{cases} \qquad \psi_2(t) = \begin{cases} \sqrt{2/T} & T/2 \le t \le T \\ 0 & \text{otherwise} \end{cases} \tag{1.1}$$

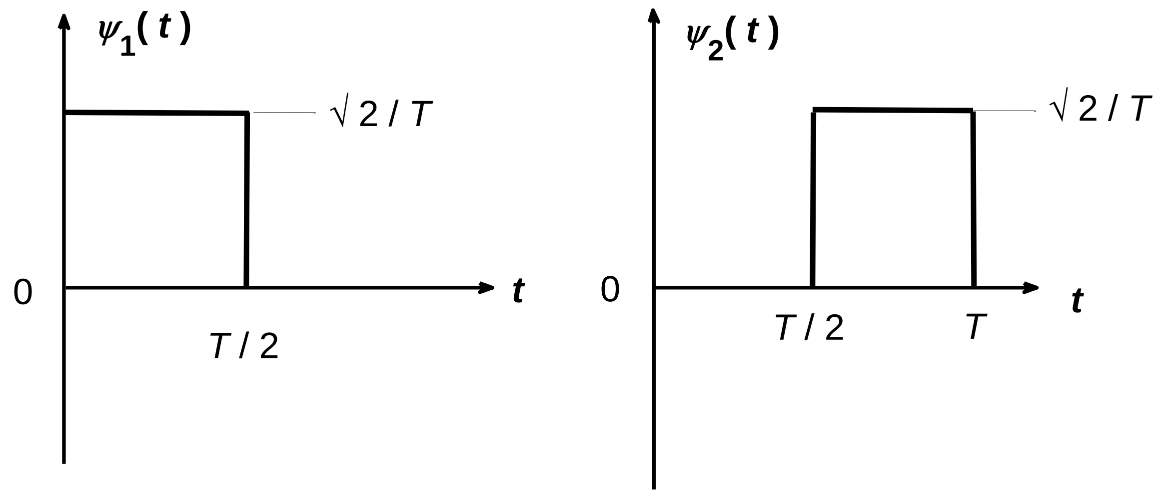

Fig. 1.2 The orthonormalized basis functions for Q1.

By using (1.1), Figs. 1.1 and 1.2, we obtain the followings for the time waveforms of $s_1(t)$ to $s_8(t)$

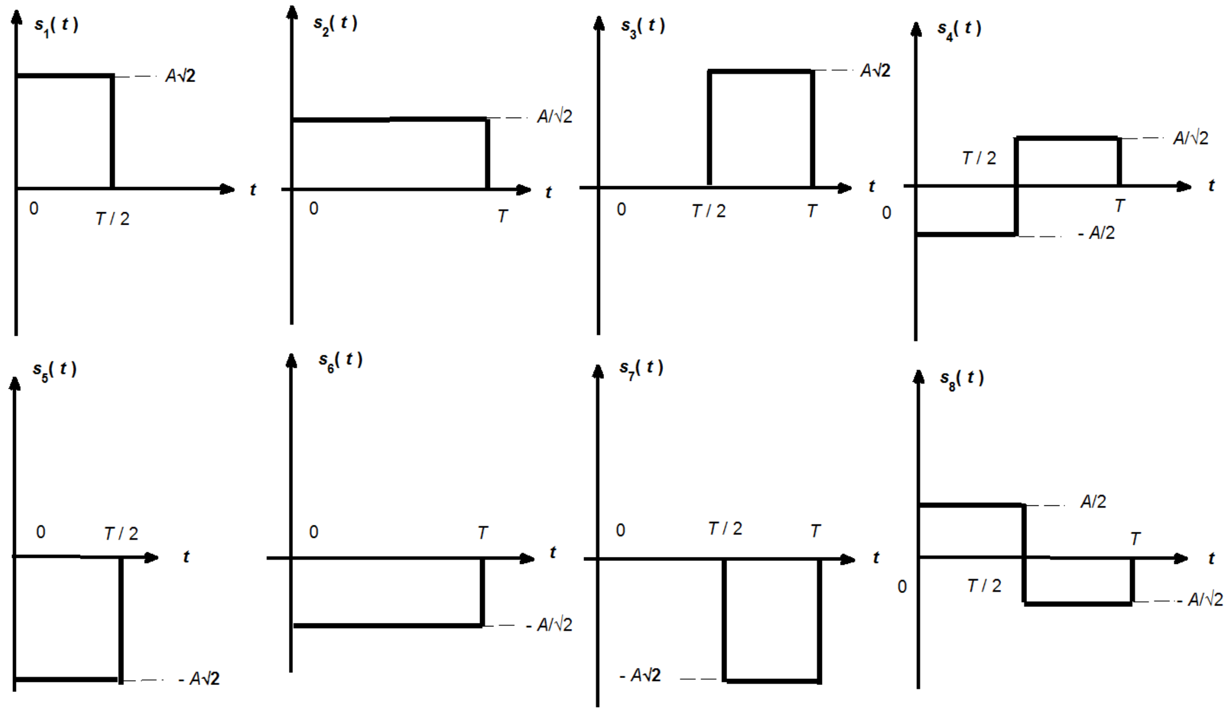

Fig. 1.3 Time waveforms of $s_1(t)\cdots s_8(t)$ for Q1

From Fig. 1.3, it is possible to write the following expressions for $s_1(t)$ to $s_8(t)$

$$s_1(t)=\begin{cases}A\sqrt{2} & 0\le t\le T/2\\ 0 & \text{otherwise}\end{cases},\; s_1(t)=A\sqrt{T}\psi_1(t),\; \mathbf{s_1}=[s_{11},\, s_{12}]=\left[A\sqrt{T},\, 0\right],\; \varepsilon_{s_1}=\|\mathbf{s_1}\|^2=A^2T$$

$$s_2(t)=\begin{cases}\dfrac{A}{\sqrt{2}} & 0\le t\le T\\ 0 & \text{otherwise}\end{cases},\; s_2(t)=\frac{A}{2}\sqrt{T}\psi_1(t)+\frac{A}{2}\sqrt{T}\psi_2(t),\; \mathbf{s_2}=[s_{21},\, s_{22}]=\left[\frac{A}{2}\sqrt{T},\, \frac{A}{2}\sqrt{T}\right],\; \varepsilon_{s_2}=\|\mathbf{s_2}\|^2=\frac{A^2T}{2}$$

$$s_3(t)=\begin{cases}A\sqrt{2} & T/2\le t\le T\\ 0 & \text{otherwise}\end{cases},\; s_3(t)=A\sqrt{T}\psi_2(t),\; \mathbf{s_3}=[s_{31},\, s_{32}]=\left[0,\, A\sqrt{T}\right],\; \varepsilon_{s_3}=\|\mathbf{s_3}\|^2=A^2T$$

$$s_4(t)=\begin{cases}-\dfrac{A}{\sqrt{2}} & 0\le t\le T/2\\ \dfrac{A}{\sqrt{2}} & T/2\le t\le T\\ 0 & \text{otherwise}\end{cases},\; s_4(t)=-\frac{A}{2}\sqrt{T}\psi_1(t)+\frac{A}{2}\sqrt{T}\psi_2(t),\; \mathbf{s_4}=[s_{41},\, s_{42}]=\left[-\frac{A}{2}\sqrt{T},\, \frac{A}{2}\sqrt{T}\right],\; \varepsilon_{s_4}=\|\mathbf{s_4}\|^2=\frac{A^2T}{2}$$

$$s_5(t)=\begin{cases}-A\sqrt{2} & 0\le t\le T/2\\ 0 & \text{otherwise}\end{cases},\; s_5(t)=-A\sqrt{T}\psi_1(t),\; \mathbf{s_5}=[s_{51},\, s_{52}]=\left[-A\sqrt{T},\, 0\right],\; \varepsilon_{s_5}=A^2T$$

$$s_6(t)=\begin{cases}-\dfrac{A}{\sqrt{2}} & 0\le t\le T\\ 0 & \text{otherwise}\end{cases},\; s_6(t)=-\frac{A}{2}\sqrt{T}\psi_1(t)-\frac{A}{2}\sqrt{T}\psi_2(t),\; \mathbf{s_6}=[s_{61},\, s_{62}]=\left[-\frac{A}{2}\sqrt{T},\, -\frac{A}{2}\sqrt{T}\right],\; \varepsilon_{s_6}=\frac{A^2T}{2}$$

$$s_7(t)=\begin{cases}-A\sqrt{2} & T/2\le t\le T\\ 0 & \text{otherwise}\end{cases},\; s_7(t)=-A\sqrt{T}\psi_2(t),\; \mathbf{s_7}=[s_{71},\, s_{72}]=\left[0,\, -A\sqrt{T}\right],\; \varepsilon_{s_7}=A^2T$$

$$s_8(t)=\begin{cases}\dfrac{A}{\sqrt{2}} & 0\le t\le T/2\\ -\dfrac{A}{\sqrt{2}} & T/2\le t\le T\\ 0 & \text{otherwise}\end{cases},\; s_8(t)=\frac{A}{2}\sqrt{T}\psi_1(t)-\frac{A}{2}\sqrt{T}\psi_2(t),\; \mathbf{s_8}=[s_{81},\, s_{82}]=\left[\frac{A}{2}\sqrt{T},\, -\frac{A}{2}\sqrt{T}\right],\; \varepsilon_{s_8}=\frac{A^2T}{2}$$

$$d_{12}=d_{18}=d_{23}=d_{34}=d_{45}=d_{56}=d_{78}=d_{\min}=A\sqrt{\frac{T}{2}}=\sqrt{\varepsilon_{s_2}}=\sqrt{\varepsilon_{s_4}}=\sqrt{\varepsilon_{s_6}}=\sqrt{\varepsilon_{s_8}}$$

$$d_{13}=d_{35}=d_{57}=d_{17}=A\sqrt{2T}=\sqrt{2\varepsilon_{s_1}}\;,\; d_{24}=d_{46}=d_{68}=d_{82}=A\sqrt{T}=\sqrt{\varepsilon_{s_1}}$$

$$d_{15}=d_{37}=d_{\max}=2A\sqrt{T}=2\sqrt{\varepsilon_{s_1}}\;, d_{25}=d_{85}=d_{14}=d_{16}=\frac{A\sqrt{10T}}{2}$$

$$|\mathbf{s}_1|=|\mathbf{s}_3|=|\mathbf{s}_5|=|\mathbf{s}_7|=A\sqrt{T}\quad,\quad |\mathbf{s}_2|=|\mathbf{s}_4|=|\mathbf{s}_6|=|\mathbf{s}_8|=A\sqrt{\frac{T}{2}}\qquad(1.2)$$

b. Since QAM is two dimensional, for block diagrams of correlator and MF, we benefit from Fig. 6.7 of ECE376_ Dimensionality of Signals_ASK_PSK_QAM_FSK_Jan 2013_HTE, which is not reproduced here to save space. Below we just give the outputs.

If $s_1\ t$ is sent from the transmitter in then the output from the upper and lower braches of the correlator will be after sampling will be

$$y_1=\int_0^T r(t)\ \psi_1(t)dt=\int_0^T s_1(t)\ \psi_1(t)dt+\int_0^T n(t)\ \psi_1(t)dt=s_{11}+n_1=A\sqrt{T}+n_1$$
$$y_2=\int_0^T r(t)\ \psi_2(t)dt=\int_0^T s_1(t)\ \psi_2(t)dt+\int_0^T n(t)\ \psi_2(t)dt=s_{12}+n_2=0+n_2\qquad(1.3)$$

We know that output from MF will be identical to (1.3) at the time of sampling at $t=T$ which means that we can construct the received vector $\mathbf{r}$ that we supply to the detector and which will be used in the decision making process, as follows

$$\mathbf{r}=\begin{bmatrix} r_1 \\ r_2 \end{bmatrix}=\begin{bmatrix} A\sqrt{T}+n_1 \\ n_2 \end{bmatrix} \quad (1.4)$$

c. Using (1.4), we evaluate correlation metrics $C\ \ \mathbf{r}, \mathbf{s}_m$ for $m=1\cdots 8$ as follows

$$C(\mathbf{r}, \mathbf{s_m})=2\,\mathbf{s_m}\cdot\mathbf{r}-\|\mathbf{s_m}\|^2\ ,\quad m=1\cdots 8$$

$$m=1\ ,\ C(\ \mathbf{r}, \mathbf{s_1})=2\,\mathbf{s_1}\cdot\mathbf{r}-\|\mathbf{s_1}\|^2=2\left[A\sqrt{T},\ 0\right]\begin{bmatrix} A\sqrt{T}+n_1 \\ n_2 \end{bmatrix}-A^2T=A^2T+2An_1\sqrt{T}$$

$$m=2\ ,\ C(\ \mathbf{r}, \mathbf{s_2})=2\,\mathbf{s_2}\cdot\mathbf{r}-\|\mathbf{s_2}\|^2=2\left[\frac{A}{2}\sqrt{T},\ \frac{A}{2}\sqrt{T}\right]\begin{bmatrix} A\sqrt{T}+n_1 \\ n_2 \end{bmatrix}-\frac{A^2T}{2}=\frac{A\sqrt{T}}{2}\left(2n_1+2n_2+A\sqrt{T}\right)$$

$$m=3\ ,\ C(\ \mathbf{r}, \mathbf{s_3})=2\,\mathbf{s_3}\cdot\mathbf{r}-\|\mathbf{s_3}\|^2=2\left[0,\ A\sqrt{T}\right]\begin{bmatrix} A\sqrt{T}+n_1 \\ n_2 \end{bmatrix}-A^2T=-A^2T+2An_2\sqrt{T}$$

$$m=4\ ,\ C(\ \mathbf{r}, \mathbf{s_4})=2\,\mathbf{s_4}\cdot\mathbf{r}-\|\mathbf{s_4}\|^2=2\left[-\frac{A}{2}\sqrt{T},\ \frac{A}{2}\sqrt{T}\right]\begin{bmatrix} A\sqrt{T}+n_1 \\ n_2 \end{bmatrix}-\frac{A^2T}{2}=-\frac{A\sqrt{T}}{2}\left(2n_1-2n_2+3A\sqrt{T}\right)$$

$$m=5\ ,\ C(\ \mathbf{r}, \mathbf{s_5})=2\,\mathbf{s_5}\cdot\mathbf{r}-\|\mathbf{s_5}\|^2=2\left[-A\sqrt{T},\ 0\right]\begin{bmatrix} A\sqrt{T}+n_1 \\ n_2 \end{bmatrix}-A^2T=-3A^2T+2An_1\sqrt{T}$$

$$m=6\ ,\ C(\ \mathbf{r}, \mathbf{s_6})=2\,\mathbf{s_6}\cdot\mathbf{r}-\|\mathbf{s_6}\|^2=2\left[0,\ -A\sqrt{T}\right]\begin{bmatrix} A\sqrt{T}+n_1 \\ n_2 \end{bmatrix}-\frac{A^2T}{2}=\frac{A\sqrt{T}}{2}\left(2n_1+2n_2+3A\sqrt{T}\right)$$

$$m=7\ ,\ C(\ \mathbf{r}, \mathbf{s_7})=2\,\mathbf{s_7}\cdot\mathbf{r}-\|\mathbf{s_7}\|^2=2\left[0,\ -\frac{A}{2}\sqrt{T}\right]\begin{bmatrix} A\sqrt{T}+n_1 \\ n_2 \end{bmatrix}-A^2T=-A^2T-2An_2\sqrt{T}$$

$$m=8\ ,\ C(\ \mathbf{r}, \mathbf{s_8})=2\,\mathbf{s_8}\cdot\mathbf{r}-\|\mathbf{s_8}\|^2=2\left[\frac{A}{2}\sqrt{T},\ -\frac{A}{2}\sqrt{T}\right]\begin{bmatrix} A\sqrt{T}+n_1 \\ n_2 \end{bmatrix}-\frac{A^2T}{2}=\frac{A\sqrt{T}}{2}\left(2n_1-2n_2+A\sqrt{T}\right) \quad (1.6)$$

The correct decision region for $\mathbf{s_1}$ for the case of $s_1(t)$ being transmitted is determined by the following inequalities and corresponding conditions.

$$C(\mathbf{r},\mathbf{s}_1) > C(\mathbf{r},\mathbf{s}_2)\ :\ \frac{A}{2}\sqrt{T}\left(\overbrace{n_1 + A\sqrt{T}}^{r_1} + n_1 - 2\overset{r_2}{\underset{r_2}{n_2}}\right) > 0 \ \rightarrow r_2 < r_1 - \frac{A}{2}\sqrt{T}$$

$$C(\mathbf{r},\mathbf{s}_1) > C(\mathbf{r},\mathbf{s}_3)\ :\ 2A\sqrt{T}\left(\overbrace{n_1 + A\sqrt{T}}^{r_1} - \overset{r_2}{\underset{r_2}{n_2}}\right) > 0 \ \rightarrow r_2 < r_1$$

$$C(\mathbf{r},\mathbf{s}_1) > C(\mathbf{r},\mathbf{s}_4)\ :\ \frac{A}{2}\sqrt{T}\left(6n_1 - 2n_2 + 5A\sqrt{T}\right) > 0 \ \rightarrow r_2 < \frac{6r_1 - A\sqrt{T}}{2}$$

$$C(\mathbf{r},\mathbf{s}_1) > C(\mathbf{r},\mathbf{s}_5)\ :\ 4A^2T + 4An_1\sqrt{T} > 0 \ \rightarrow r_1 > 0$$

$$C(\mathbf{r},\mathbf{s}_1) > C(\mathbf{r},\mathbf{s}_6)\ :\ \frac{A}{2}\sqrt{T}\left(6n_1 + 2n_2 + 5A\sqrt{T}\right) > 0 \ \rightarrow r_2 > -\frac{6r_1 - A\sqrt{T}}{2}$$

$$C(\mathbf{r},\mathbf{s}_1) > C(\mathbf{r},\mathbf{s}_7)\ :\ 2A\sqrt{T}\left(n_1 + A\sqrt{T} + n_2\right) > 0 \ \rightarrow r_2 > -r_1$$

$$C(\mathbf{r},\mathbf{s}_1) > C(\mathbf{r},\mathbf{s}_8)\ :\ \frac{A}{2}\sqrt{T}\left(n_1 + A\sqrt{T} + n_1 + 2n_2\right) > 0 \ \rightarrow r_2 > -r_1 + \frac{A}{2}\sqrt{T} \qquad (1.7)$$

The computations of (1.6) and (1.7) are in ECE376_MT2017_Q1Calculations.m.

From (1.7), we need to take into account only the following inequalities

$$C(\mathbf{r},\mathbf{s}_1) > C(\mathbf{r},\mathbf{s}_2) \ \text{ and } \ C(\mathbf{r},\mathbf{s}_1) > C(\mathbf{r},\mathbf{s}_8) \qquad (1.8)$$

since the other inequalities are already covered by the ones given in (1.8).

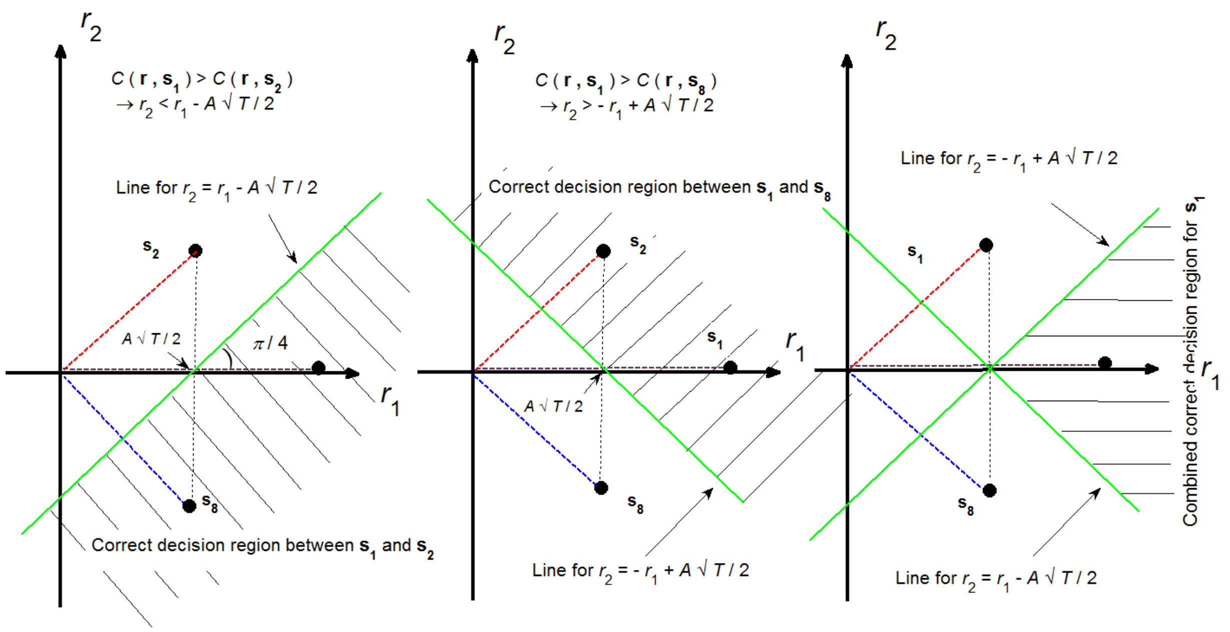

Fig. 1. 4 Decision regions for $\mathbf{s}_1$ against $\mathbf{s}_2$, $\mathbf{s}_8$ and combined correct decision region for $\mathbf{s}_1$.

To find the probability of error for $\mathbf{s}_1$, we need to integrate in the shaded area of Fig. 1.4, which is left as an exercise.

2. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones, justify your answer

a) The demodulation of FM signal can be done by a varactor diode : False, the varactor diode is used to obtain FM as explained on pp. 12-13 of lecture notes entitled, " ECE 376_AM_FM Demodulation_Jan 2013_HTE".

b) In envelope detection of full AM, we use a diode and a low pass filter : True, as explained on pp. 5-7 of lecture notes entitled, " ECE 376_AM_FM Demodulation_Jan 2013_HTE".

c) For demodulation, we use a local carrier at the receiver which is phase synchronized to the one sent from the transmitter : This is one way of performing demodulation, but this is not necessary for full AM as given above.

d) PSK is multidimensional : False, PSK is two dimensional as explained on pp. 12-13 of lecture notes entitled, " ECE376_ Dimensionality of Signals_ASK_PSK_QAM_FSK_Jan 2013_HTE".

e) As the number of dimensionality of the signal increases, the bandwidth requirement also increases : True, as explained on pp. 22 of lecture notes entitled, " ECE376_ Dimensionality of Signals_ASK_PSK_QAM_FSK_Jan 2013_HTE".

f) The constellation diagram of QAM contains 8 signal vectors : False, it can take on any Mary value.