Çankaya University – ECE Department – ECE 376

Student Name : Open source exam
Student Number : Duration : 2 hours

Questions

1. (30 Points) 155 Mbits/sec binary message signal is given. We plan to hire a channel with a bandwidth 10 MHz to pass this binary message signal. Find into what Mary PSK or QAM we should use for this purpose. The 10 MHz channel is measured to be flat in slices of 1 MHz. Hence we further plan to convert this Mary PSK or QAM signal into OFDM. Find the subcarriers in such an OFDM system.

Solution : Assuming the inverse of the bit/symbol duration is approximately equal to the required bandwidth, then for a $R = 155$ Mbits/sec , binary message signal, we would require a bandwidth of

$$B_b = R = 1/T_b = 155 \text{ MHz} \quad , \quad T_b = 6.45 \text{ nsec} \tag{1.1}$$

It is given that $B_c = 10$ MHz , we need to set (symbol duration) $T_s = 1/10 \text{ MHz} = 100$ nsec . Thus converting to M ary PSK or QAM, we should have $k = T_s / T_b = 15.5 \rightarrow 16 = \log_2 M \;\; \rightarrow \;\; M = 65536$.

By setting the number of subcarriers to the M ary level, we calculate the symbol duration for the individual subcarriers as

$$T = MT_s = 65536 T_s = 6.5 \text{ msec} \tag{1.2}$$

For orthogonality to be satisfied whilst remaining within the 10 MHz given bandwidth, our subcarriers must have an integer number of cycles within $T = 6.5$ msec . Hence a simple choice is

$$f_1 = \frac{1}{T} = 152.5879 \text{ Hz} \; , \; f_2 = \frac{2}{T} = 305.1758 \text{ Hz} \; , \; \cdots f_n = \frac{n}{T} = n \times 152.5879 \text{ Hz} \; , \; \cdots \; , \; f_{65536} = 10 \text{ MHz} \tag{1.3}$$

For the frequencies listed in (1.3), it is easy to verify that

$$f_p - f_m = \frac{p-m}{T} \quad , \quad 1 \le m < M \quad , \quad p = m + i \quad , \quad 1 \le i < M - m \tag{1.4}$$

Note that in the frequency listing of (1.3), both the given channel bandwidth is not exceeded, additionally adjacent frequency spacing is less than the 1 MHz slices of the total channel bandwidth, namely

$$f_{65536} - f_1 = 9.9998 \text{ MHz} < B_c = 10 \text{ MHz} \quad , \quad f_p - f_{p-1} = 152.5879 \text{ Hz} << 1 \text{ MHz} \tag{1.5}$$

We can check for orthogonality, both by assigning sinusoidal and exponential carriers. This is done below

$$I = \int_0^T \cos\left(2\pi f_p t\right)\cos\left(2\pi f_m t\right) dt \quad , \quad I_e = \int_0^T \exp\left(j2\pi f_p t\right)\exp\left(-j2\pi f_m t\right) dt \tag{1.6}$$

(1.6) can be verified in Matlab quite easily.

A few comments are in order for the OFDM system design of above. One is number of subcarriers are too high, if a practical implementation becomes difficult with such number of subcarriers, possible solutions are

a) Increase the bandwidth above 10 MHz
b) Use a smoother shaping waveform, thus have a reduction in Mary value

2. (40 Points) Two spreading sequences of $L_c =$ are given below

$$\mathbf{e}_1 = \left[-1\,1\,1\ -1\ -1\,1\,1\right] \quad , \quad \mathbf{e}_2 = \left[-1\,1\ -1\ -1\,1\ -1\ -1\right] \tag{2.1}$$

We have two message signals using the spreading sequences of (2.1). Assuming that in the time interval 0 to $T_b = 0.2$ msec the of the first user is ${}_1a_0 = +1$, while for the second user ${}_2a_0 = -1$. Following the operations in Fig.2.2 of lecture notes entitled, "Spread spectrum systems_2013_HTE", find the outputs of the correlators, z_1 and z_2. Make comments on your findings.

Solution : The relevant block diagram is reproduced below.

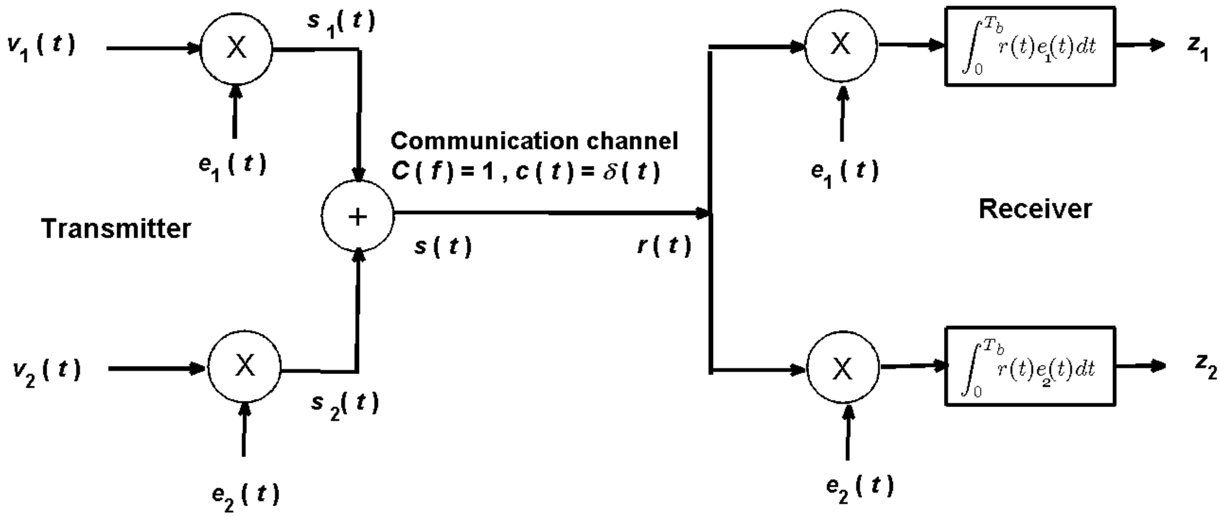

Fig. 2.1 Block diagram of a DS system consisting of two users.

To evaluate z_1 and z_2, we use (2.17) and (2.18) of lecture notes entitled, "Spread spectrum systems_2013_HTE", which are

$$z_1 = {}_1a_0 T_b + {}_2a_0 \int_0^{T_b} e_1(t)\, e_2(t)\, dt = T_b \left[{}_1a_0 + {}_2a_0 \frac{1}{T_b} \int_0^{T_b} e_1(t)\, e_2(t)\, dt \right]$$

$$z_2 = {}_2a_0 T_b + {}_1a_0 \int_0^{T_b} e_1(t)\, e_2(t)\, dt = T_b \left[{}_2a_0 + {}_1a_0 \frac{1}{T_b} \int_0^{T_b} e_1(t)\, e_2(t)\, dt \right] \tag{2.1}$$

From the same notes, we know that

$$\frac{1}{T_b}\int_0^{T_b} e_1(t)\, e_2(t)\, dt \rightarrow \mathbf{e_1}\ \mathbf{e_2^t}\begin{bmatrix}-1 & 1 & 1 & -1 & -1 & 1 & 1\end{bmatrix}\begin{bmatrix}-1\\1\\-1\\-1\\1\\-1\\-1\end{bmatrix} = \frac{-1}{7} \qquad (2.2)$$

Hence after inserting the numeric values, (2.1) will become

$$z_1 = 2\times10^{-4}\left(1+\frac{1}{7}\right) \quad , \quad z_2 = -2\times10^{-4}\left(1+\frac{1}{7}\right) \qquad (2.3)$$

It is quite easy to see that

$$\text{Sign}(z_1) = \text{Sign}\left[2\times10^{-4}\left(1+\frac{1}{7}\right)\right] \rightarrow 1 \text{ , } \text{Sign}(z_2) = \text{Sign}\left[-2\times10^{-4}\left(1+\frac{1}{7}\right)\right] \rightarrow -1 \qquad (2.4)$$

So that demodulation will be successful.

3. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones justify your answer.

a) DS spread spectrum systems employ PN sequences with large L_c : True, since we need quite maximum number of PN sequences, which can be achieved by using large L_c .

b) In code division multiplexing, we have to use PN sequences, which are orthogonal : True, this is the essential requirement for PN sequences.

c) In QAM, there is no energy difference between the signal vectors : False, since QAM is a combination of PSK and ASK.

d) OFDM uses orthogonal subcarriers to improve the error performance : Incomplete, the use of orthogonal subcarriers also conserves on bandwidth, as shown in Fig. 3.6 of lecture notes entitled, "Notes on OFDM_2013".

e) FM is a type of spread spectrum system : It can be considered as such since after modulation infinite number of sidebands are created. But there is no philosophy of CDM in FM.