Çankaya University – ECE Department – ECE 376

Student Name :
Student Number :

Open source exam
Duration : 2 hours

Questions

1. (40 Points) Assume that we have a 16 Mbits/sec binary message signal. This signal is to be transmitted using M ary PSK or QAM over a channel which has 4 MHz bandwidth. Determine what M ary level should be used here in order to pass 16 Mbits/sec signal through this channel. Now assume that we decide to feed the PSK or QAM signals to OFDM modulator and use OFDM for the same channel. Setting the number of subcarriers of OFDM to the M ary number of PSK or QAM, determine frequencies of subcarriers and check for their orthogonality.

Solution : Assuming the inverse of the symbol duration is approximately equal to the required bandwidth, then for a 4 MHz bandwidth, we have $T_s = 1/4\ \mu\text{sec}$. For 16 Mbits/sec signal on the other hand $T_b = 1/16\ \mu\text{sec}$, then $k = T_s / T_b = 4 = \log_2 M \ \rightarrow\ M = 16$. Thus M ary level has to 16.

Since number of subcarriers is to be set to M ary level, we calculate the symbol duration for the individual subcarriers as

$$T = MT_s = 16T_s = 4\ \mu\text{sec} \tag{1.1}$$

For orthogonality to be satisfied whilst remaining within the 4 MHz given bandwidth, our subcarriers must have an integer number of cycles within $T = 4\ \mu\text{sec}$. Hence a suitable choice is

$$\begin{gathered}
f_1 = \frac{1}{T} = 0.25 \text{ MHz}\ ,\ f_2 = \frac{2}{T} = 0.5 \text{ MHz}\ ,\ f_3 = \frac{3}{T} = 0.75 \text{ MHz}\ ,\ f_4 = \frac{4}{T} = 1 \text{ MHz} \\
f_5 = \frac{5}{T} = 1.25 \text{ MHz}\ ,\ f_6 = \frac{6}{T} = 1.5 \text{ MHz}\ ,\ f_7 = \frac{7}{T} = 1.75 \text{ MHz}\ ,\ f_8 = \frac{8}{T} = 2 \text{ MHz} \\
f_9 = \frac{9}{T} = 2.25 \text{ MHz}\ ,\ f_{10} = \frac{10}{T} = 2.5 \text{ MHz}\ ,\ f_{11} = \frac{11}{T} = 2.75 \text{ MHz}\ ,\ f_{12} = \frac{12}{T} = 3 \text{ MHz} \\
f_{13} = \frac{13}{T} = 3.25 \text{ MHz}\ ,\ f_{14} = \frac{14}{T} = 3.5 \text{ MHz}\ ,\ f_{15} = \frac{15}{T} = 3.75 \text{ MHz}\ ,\ f_{16} = \frac{16}{T} = 4 \text{ MHz}
\end{gathered} \tag{1.2}$$

For the list of frequencies in (1.2), it is easy to verify that

$$f_p - f_m = \frac{p-m}{T}\ \ ,\ \ 1 \le m < M\ \ ,\ \ p = m + i\ \ ,\ \ 1 \le i < M - m \tag{1.3}$$

We can check for orthogonality, both by assigning sinusoidal and exponential carriers. This is done below

$$I = \int_0^T \cos\left(2\pi f_p t\right)\cos\left(2\pi f_m t\right)dt \quad , \quad I_e = \int_0^T \exp\left(j2\pi f_p t\right)\exp\left(-j2\pi f_m t\right)dt \tag{1.4}$$

The first is done by the following Matlab code, also available in Q1ort_FE_30052014.m.

```
clear;clc ;clf reset;close all
syms t
T = 4;fsub = 0.25:0.25:4;M = 16;Iarr = [];
for m = 1:M-1
    for i = 1:M - m
        p = m + i;
c1 = cos(2*pi*fsub(p)*t);c2 = cos(2*pi*fsub(m)*t);
I = int(c1*c2,t,0,T);Iarr = [Iarr I];
end;end
if Iarr == 0;disp('OK');end
```

```
The answer is OK
```

The test for exponential subcarriers is carried by slightly modifying the above code as

```
clear;clc ;clf reset;close all
syms t
T = 4;fsub = 0.25:0.25:4;M = 16;Iarr = [];Iearr = [];
for m = 1:M-1
    for i = 1:M - m
        p = m + i;
c1 = cos(2*pi*fsub(p)*t);c2 = cos(2*pi*fsub(m)*t);
c1e = exp(j*2*pi*fsub(p)*t);c2e = exp(-j*2*pi*fsub(m)*t);
I = int(c1*c2,t,0,T);Iarr = [Iarr I];
I = int(c1e*c2e,t,0,T);Iearr = [Iearr I];
end;end
if Iarr == 0;disp('OK');end
if Iearr == 0;disp('OK');end
```

```
The answers are OK
```

2. (30 Points) A message signal of 2 kbits/sec is to be transmitted using code division multiplexing, where the spreading waveform (PN sequence) has $L_c = 511$. Plot the time waveforms and frequency spectrums of message signal, PN sequence and modulated waveform. Write for the corresponding time waveform expressions. Draw an appropriate receiver diagram for this case. Specify the requirements for the PN sequence to be used at receiver.

Solution : By defining the time waveform expressions and frequency spectrums of the message signal, PN sequence and the modulated waveform as

$$\text{Message signal : } v(t) = \sum_{n=-\infty}^{\infty} a_n g(t - nT_b) \quad , \quad g(t) = \begin{cases} T_b^{-0.5} & 0 \le t \le T_b \\ 0 & \text{otherwise} \end{cases} \ , \ T_b = 0.5 \text{ msec}$$

$$\text{PN sequence : } e(t) = \sum_{i=0}^{L_c-1} e_i p(t - iT_c) \ \ 0 \le t \le T_b \ , \ p(t) = \begin{cases} T_c^{-0.5} & 0 \le t \le T_c \\ 0 & \text{otherwise} \end{cases} \ , \ T_c = T_b / L_c = 0.97 \ \mu\text{sec}$$

$$\text{Modulated (spread) signal: } s(t) = v(t)e(t) == \sum_{n=-\infty}^{\infty} a_n g(t - nT_b) \sum_{i=0}^{L_c-1} e_i p(t - iT_c)$$

$$V(f) = \mathbf{F}\left[v(t)\right] \ , E(f) = \mathbf{F}\left[e(t)\right] \ , \ S(f) = \mathbf{F}\left[s(t)\right] \qquad (1.1)$$

We illustrate in Figs 2.1 and 2.2 the time waveforms and the spectrums of these expressions given in (2.1)

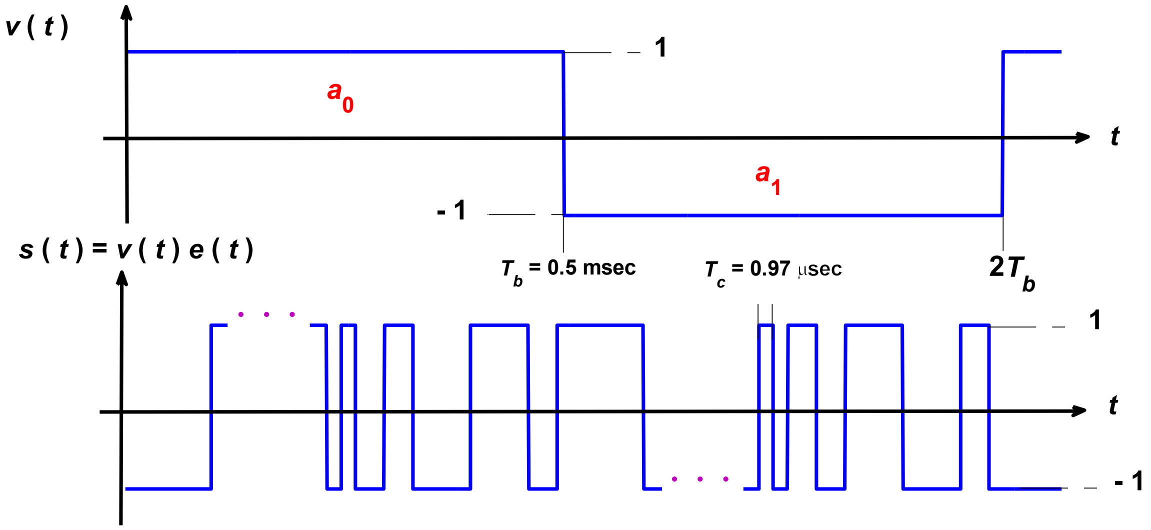

Fig. 2.1 Time waveforms of the message and the modulated signals.

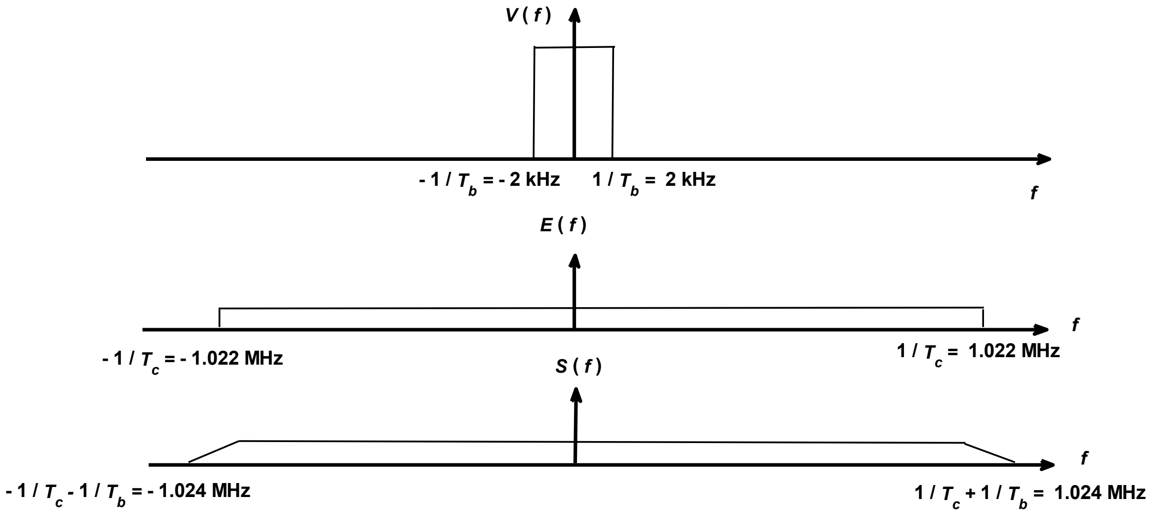

Fig. 2.2 Approximate frequency spectrums of the message signal, PN sequence and the modulated signal.

An appropriate receiver diagram incorporating a correlator is given in Fig. 2.3.

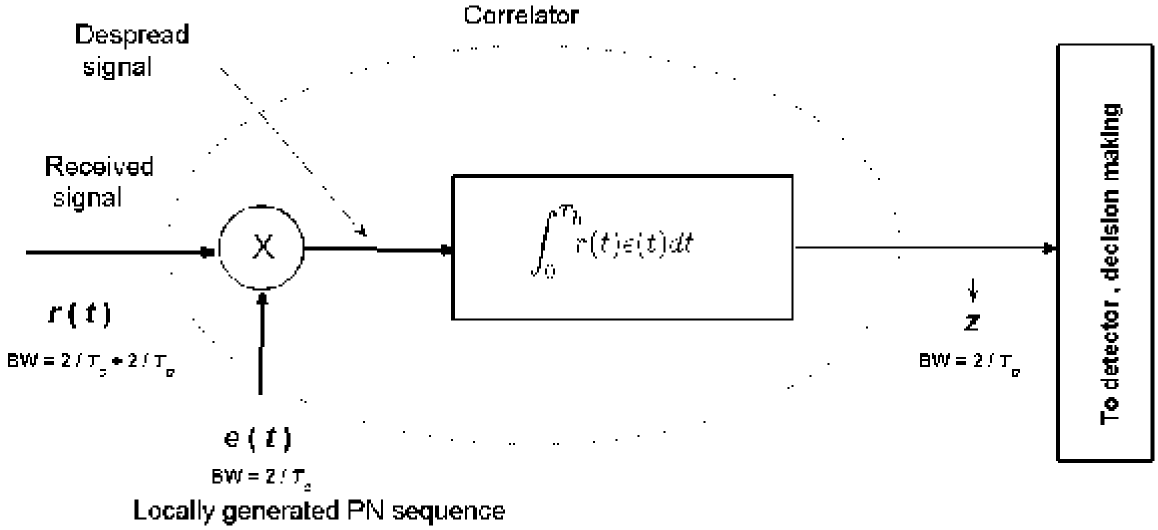

Fig. 2.3 An appropriate receiver diagram for the demodulation of the spread signal.

As seen from Fig. 2.3, at the receiver side, the PN sequence is denoted as $e(t)$, which means that we have somehow managed time synchronization between the transmitter and receiver. This is the most essential requirement.

3. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones justify your answer.

a) In code division multiplexing, we use codes : This is misleading description, a better wording is “In code division multiplexing, we use spreading codes to spread the message signal from narrow band to wide band”.

b) In OFDM, we decide on the number of subcarriers by taking into the Mary level of PSK or QAM input signal : Although this has been the case in our notes, examples and exam questions, in practical implementation, this is decided by the amount of nonflatness in the communication channel used.

c) In OFDM, the subcarriers are multiplexed along time axis : False, the subcarriers are added along time axis.

d) In spread spectrum systems, spreading is done along time axis : False, in spread spectrum systems, spreading is done along frequency axis as seen from Fig. 2.2 of above. This is achieved by slicing the message signal along time axis, as also seen from Fig. 2.1 of above.

e) At receiver, we do the detection by matched filters or correlators : False, if we consider that detection covers demodulation plus decision mechanism, then demodulation or obtaining the components of received signal along the respective orthonormalized basis function axis is done by matched filters or correlators, then from the numeric results of the correlator metrics, we make a decision.