Çankaya University – ECE Department – ECE 376

Student Name : Open source exam
Student Number : Duration : 2 hours

Questions

1. (35 Points) We wish to pass 100 Mbits/sec binary message signal, through a channel which has 25 MHz bandwidth. The frequency response of the channel is flat only within slices of 2 MHz. Design the modulation schemes that should be applied to this signal on the transmitter side, so that we can pass 100 Mbits/sec signal through the channel with minimum distortions in its frequency spectrum. Plot the related approximate waveforms and the frequency spectrums.

Solution : Assuming the inverse of the bit/symbol duration is approximately equal to the required bandwidth, then for a $R = 100 \text{ Mbits/sec}$, binary message signal, we would require a bandwidth of

$$B_b = R = 1/T_b = 100 \text{ MHz} \tag{1.1}$$

Since $B_c = 25 \text{ MHz}$, we need to set $T_s = 1/25 \text{ MHz} = 40 \text{ nsec}$. Thus converting to M ary PSK or QAM, we should have $k = T_s / T_b = 4 = \log_2 M \;\rightarrow\; M = 16$.

By setting the number of subcarriers to the M ary level, we calculate the symbol duration for the individual subcarriers as

$$T = MT_s = 16T_s = 0.64 \; \mu\text{sec} \tag{1.2}$$

For orthogonality to be satisfied whilst remaining within the 25 MHz given bandwidth, our subcarriers must have an integer number of cycles within $T = 0.64 \; \mu\text{sec}$. Hence a suitable choice is

$$\begin{aligned}
&f_1 = \frac{1}{T} = 1.5625 \text{ MHz}, \; f_2 = \frac{2}{T} = 3.125 \text{ MHz}, \; f_3 = \frac{3}{T} = 4.6875 \text{ MHz}, \; f_4 = \frac{4}{T} = 6.25 \text{ MHz} \\
&f_5 = \frac{5}{T} = 7.8125 \text{ MHz}, \; f_6 = \frac{6}{T} = 9.375 \text{ MHz}, \; f_7 = \frac{7}{T} = 10.9375 \text{ MHz}, \; f_8 = \frac{8}{T} = 12.5 \text{ MHz} \\
&f_9 = \frac{9}{T} = 14.0625 \text{ MHz}, \; f_{10} = \frac{10}{T} = 15.625 \text{ MHz}, \; f_{11} = \frac{11}{T} = 17.1875 \text{ MHz}, \; f_{12} = \frac{12}{T} = 18.75 \text{ MHz} \\
&f_{13} = \frac{13}{T} = 20.3125 \text{ MHz}, \; f_{14} = \frac{14}{T} = 21.875 \text{ MHz}, \; f_{15} = \frac{15}{T} = 23.4375 \text{ MHz}, \; f_{16} = \frac{16}{T} = 25 \text{ MHz}
\end{aligned} \tag{1.3}$$

For the frequencies listed in (1.3), it is easy to verify that

$$f_p - f_m = \frac{p-m}{T} \;\; , \;\; 1 \le m < M \;\; , \;\; p = m + i \;\; , \;\; 1 \le i < M - m \tag{1.4}$$

Note that in the frequency listing of (1.3), both the given channel bandwidth is not exceeded, additionally adjacent frequency spacing is less than the 2 MHz slices of the total channel bandwidth, namely

$$f_{16} - f_1 = 23.4375 \text{ MHz} < B_c = 25 \text{ MHz} \quad , \quad f_p - f_{p-1} = 1.5625 \text{ MHz} < 2 \text{ MHz} \tag{1.4}$$

We can check for orthogonality, both by assigning sinusoidal and exponential carriers. This is done below

$$I = \int_0^T \cos\left(2\pi f_p t\right)\cos\left(2\pi f_m t\right) dt \ \ , \ \ I_e = \int_0^T \exp\left(j2\pi f_p t\right)\exp\left(-j2\pi f_m t\right) dt \tag{1.5}$$

The first is done by the following Matlab code, also available in Q1ort_FE_01062015.m.

```
clear;clc ;clf reset;close all
syms t
T = 0.64;fsub = 1.5625:1.5625:25;M = 16;Iarr = [];Iearr = [];
for m = 1:M-1
    for i = 1:M - m
        p = m + i;
c1 = cos(2*pi*fsub(p)*t);c2 = cos(2*pi*fsub(m)*t);
c1e = exp(j*2*pi*fsub(p)*t);c2e = exp(-j*2*pi*fsub(m)*t);
I = int(c1*c2,t,0,T);Iarr = [Iarr I];
I = int(c1e*c2e,t,0,T);Iearr = [Iearr I];
end;end
if Iarr == 0;disp('OK');end
if Iearr == 0;disp('OK');end
```

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The answers are OK
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The time waveforms and the frequency spectrums will be similar to Figs. 3.4, 3.5 and 3.6 of Notes on OFDM_2013 and Fig. 1.1 of ECE 376_FE-03.06.2013_Solutions.

2. (35 Points) Two message signals at 10 kbits/sec are to be transmitted using code division multiplexing, where the spreading waveform (PN sequence) are

$$\mathbf{e}_1 = [-1\ -1 1 1 1\ -1 1]\ ,\ \mathbf{e}_2 = [-1\ -1 1\ -1 1 1 1] \tag{2.1}$$

Plot the time waveforms and frequency spectrums of message signal, PN sequence and modulated waveform. Write for the corresponding time waveform expressions. Draw an appropriate receiver diagram for this case.

Solution : By taking a message sequence of $v_1(t) = v_2(t) = [1\ -1]$, the relevant time waveforms and frequency spectrums of $s_1(t) = v_1(t)e_1(t)$, $S_1(f) = V_1(f) \otimes E_1(f)$, $s_2(t) = v_2(t)e_2(t)$, $S_2(f) = V_2(f) \otimes E_2(f)$ are plotted in Figs. 2.1 and 2.2.

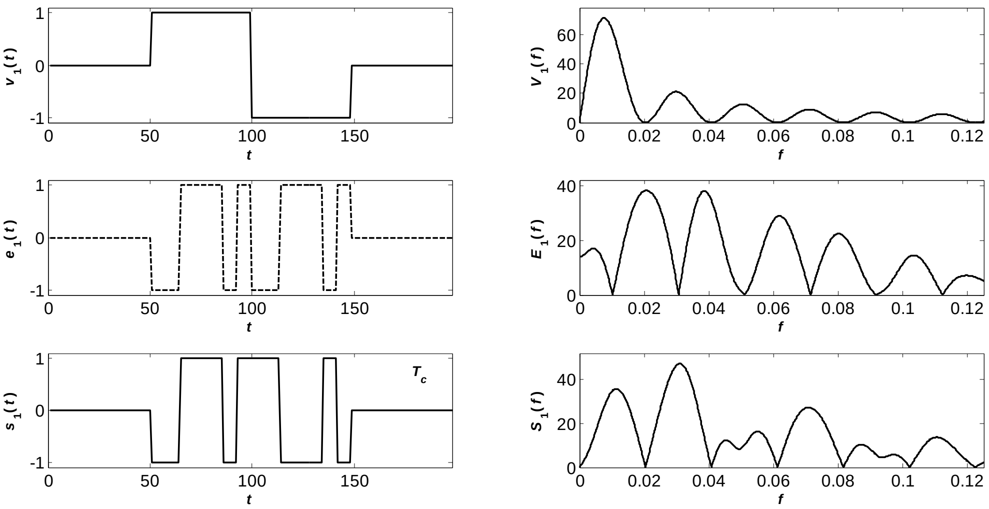

Fig. 2.1 Time waveforms and the frequency spectrums for $e_1(t)$.

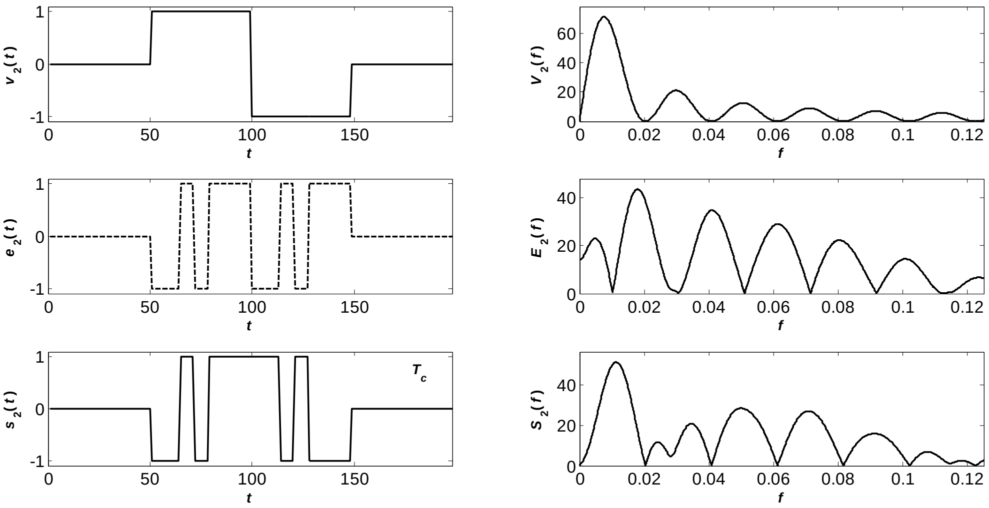

Fig. 2.2 Time waveforms and the frequency spectrums for $e_2(t)$.

The time waveform expressions are given below

$$v_1(t)=\sum_{n=-\infty}^{\infty}{}_1a_n g(t-nT_b) \quad , \quad v_2(t)=\sum_{n=-\infty}^{\infty}{}_2a_n g(t-nT_b)$$

$$g(t)=\begin{cases} T_b^{-0.5} & 0\le t\le T_b \\ 0 & \text{otherwise}\end{cases} \quad , \; T_b=0.1\text{ msec}$$

$$\text{PN sequence : } e_1(t)=\sum_{i=0}^{6}{}_1e_i p(t-iT_c) \;\; , \; e_1(t)=\sum_{i=0}^{6}{}_2e_i p(t-iT_c) \;\; , \; 0\le t\le T_b$$

$$\mathbf{e}_1=\left[-1\ -1\,1\,1\,1\ -1\,1\right] \;,\; \mathbf{e}_2=\left[-1\ -1\,1\ -1\,1\,1\,1\right]$$

$$p(t)=\begin{cases} T_c^{-0.5} & 0\le t\le T_c \\ 0 & \text{otherwise}\end{cases} \quad , \; T_c=T_b/L_c=14.3\ \mu\text{sec}$$

$$s_1(t)=v_1(t)e_1(t)=\sum_{n=-\infty}^{\infty}{}_1a_n g(t-nT_b)\sum_{i=0}^{L_C-1}{}_1e_i p(t-iT_c)$$

$$s_2(t)=v_2(t)e_2(t)=\sum_{n=-\infty}^{\infty}{}_2a_n g(t-nT_b)\sum_{i=0}^{L_C-1}{}_2e_i p(t-iT_c) \qquad (2.2)$$

An appropriate receiver diagram (including the transmitter) is shown in Fig. 2.3.

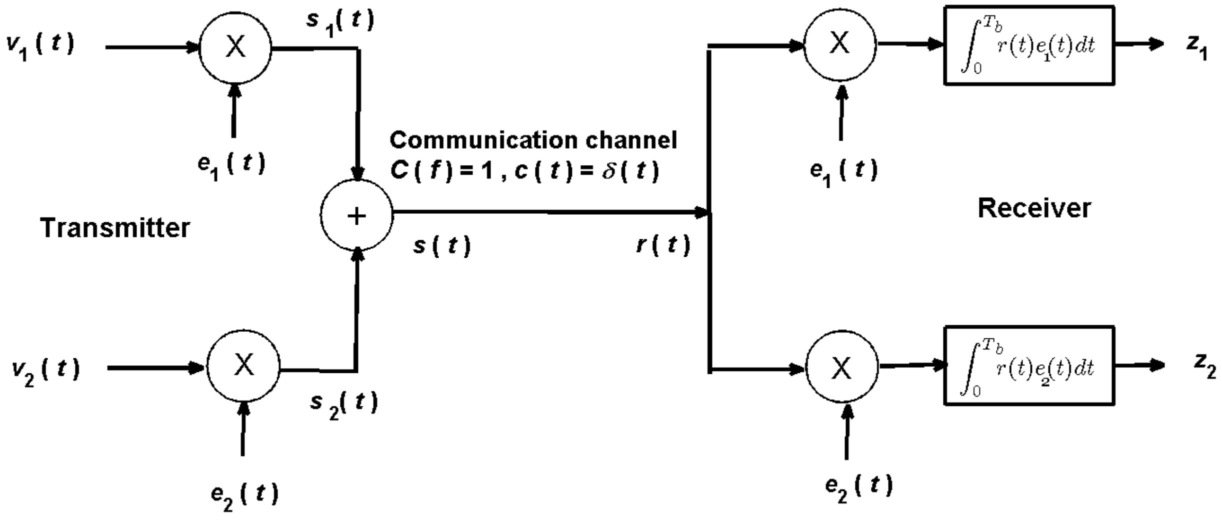

Fig. 2.3 Block diagram of a transmitter and receiver for two users.

3. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones justify your answer.

a) OFDM is used generally for atmospheric channels : True, since cable or guided channels usually have flat responses within a given bandwidth.

b) In code division multiplexing, we align message signals along frequency axis : False, in CDM, the message signal are multiplied by different PN sequences, so the alignment is along code axis.

c) In PSK, we can see amplitude variations along time axis : True as seen in the solutions of ECE 376_MT-30.03.2006 in Q1 upper figure. But symbol energies do not change.

d) CDM uses orthogonal subcarriers : False, since it is OFDM that uses orthogonal subcarriers.

e) Correlator and matched filter have the same output : True as shown in the lecture notes entitled, “ECE376_ Dimensionality of Signals_ASK_PSK_QAM_FSK_Jan 2013_HTE”