Better Constellation for Multidimensional Signals HTE - 09.03.2015

1. The case of 4 PSK (in two dimensional signal space)

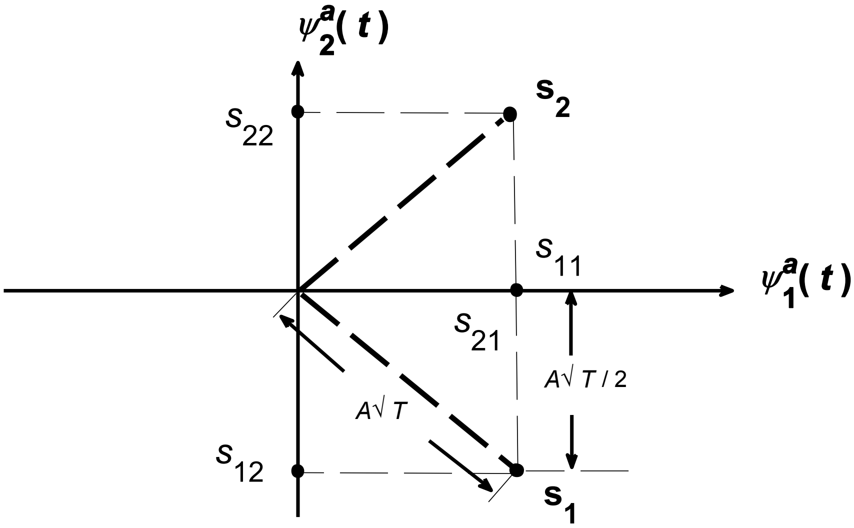

Fig. 1.1 Partial signal space diagram for two waveforms of 4 PSK.

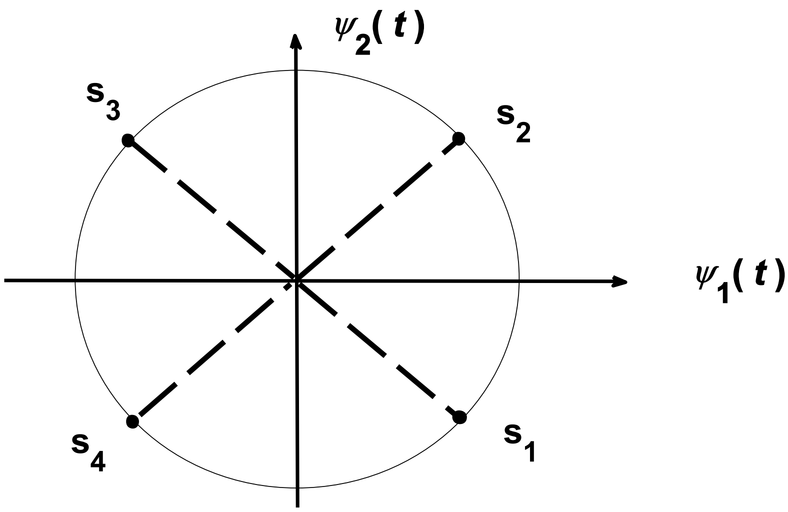

Fig. 1.2 Full signal space diagram for 4 PSK signals.

Evaluations of the minimum distance, $d_{\min 4PSK}$ for the constellations given in Figs. 1.1 and 1.2 are given in (1.1)

$$\mathbf{s}_1 = \left[\overbrace{A\sqrt{T/2}}^{\psi_1(t)} \ , \ \overbrace{-A\sqrt{T/2}}^{\psi_2(t)} \right] \ , \ \mathbf{s}_2 = \left[\overbrace{A\sqrt{T/2}}^{\psi_1(t)} \ , \ \overbrace{A\sqrt{T/2}}^{\psi_2(t)} \right] \ ,$$

$$\mathbf{s}_3 = \left[\overbrace{-A\sqrt{T/2}}^{\psi_1(t)} \ , \ \overbrace{A\sqrt{T/2}}^{\psi_2(t)} \right] \ , \ \mathbf{s}_4 = \left[\overbrace{-A\sqrt{T/2}}^{\psi_1(t)} \ , \ \overbrace{-A\sqrt{T/2}}^{\psi_2(t)} \right]$$

$$d_{\min 4PSK} = d_{12} = \left|\mathbf{s}_1 - \mathbf{s}_2\right| = \left[\left(A\sqrt{T/2} - A\sqrt{T/2} \right)^2 + \left(-A\sqrt{T/2} - A\sqrt{T/2} \right)^2 \right]^{0.5} = A\sqrt{2T} \qquad (1.1)$$

2 Orthogonal Signal Waveforms and Pulse Position Modulation (PPM)

Orthogonal signal waveforms $s_1^w(t) \cdots s_4^w(t)$ and pulse position modulation signal waveforms $s_1^p(t) \cdots s_4^p(t)$ are shown in Fig. 2.1.

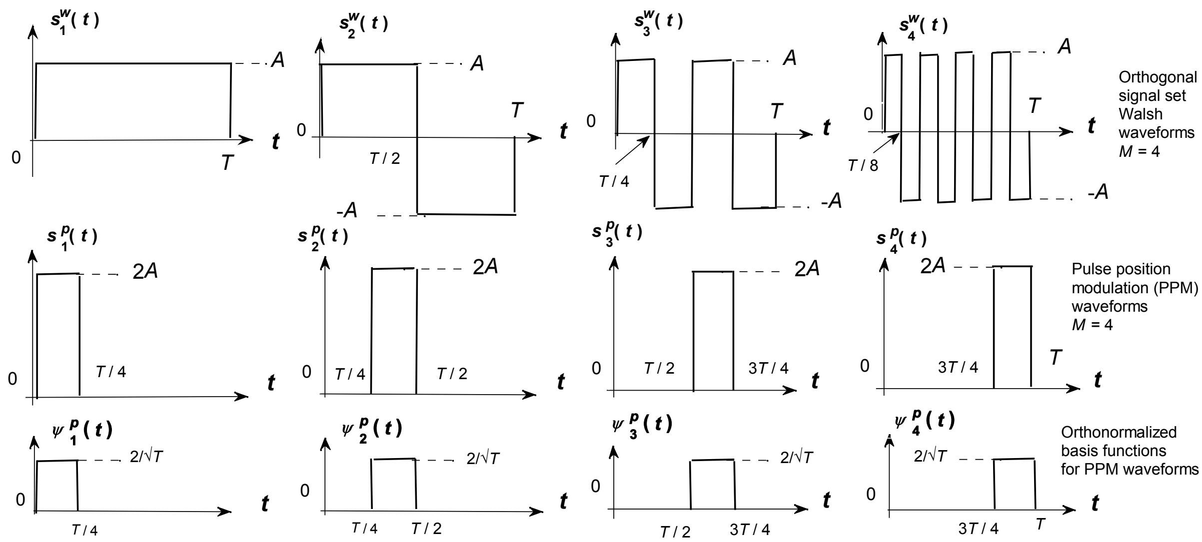

Fig. 2.1 Orthogonal signal waveform set (Walsh waveforms), pulse position modulation (PPM) waveforms and the orthonormalized basis functions for PPM waveforms.

The orthogonal waveforms displayed on the first row of Fig. 2.1 are derived from Walsh sequences and hence named as such. In both cases of Fig. 4.3, $M = N = 4$, that is the number of signals in the signal set is equivalent to the number of dimensions of the set. Therefore, we obviously need four orthonormalized basis functions to represent $s_1^w(t) \cdots s_4^w(t)$ and $s_1^p(t) \cdots s_4^p(t)$. Those for the PPM set are given on the last row of Fig. 2.1. Hence the PPM waveforms themselves, their time waveform and vectorial representation in terms of this basis functions $\psi_1^p(t) \cdots \psi_4^p(t)$ can be written as follows

$$s_1^p(t)=\begin{cases}2A & 0\le t\le T/4\\ 0 & \text{otherwise}\end{cases}\qquad s_2^p(t)=\begin{cases}2A & T/4\le t\le T/2\\ 0 & \text{otherwise}\end{cases}$$

$$s_3^p(t)=\begin{cases}2A & T/2\le t\le 3T/4\\ 0 & \text{otherwise}\end{cases}\qquad s_4^p(t)=\begin{cases}2A & 3T/4\le t\le T\\ 0 & \text{otherwise}\end{cases}$$

$$\psi_1^p(t)=\begin{cases}2/\sqrt{T} & 0\le t\le T/4\\ 0 & \text{otherwise}\end{cases}\qquad \psi_2^p(t)=\begin{cases}2/\sqrt{T} & T/4\le t\le T/2\\ 0 & \text{otherwise}\end{cases}$$

$$\psi_3^p(t)=\begin{cases}2/\sqrt{T} & T/2\le t\le 3T/4\\ 0 & \text{otherwise}\end{cases}\qquad \psi_4^p(t)=\begin{cases}2/\sqrt{T} & 3T/4\le t\le T\\ 0 & \text{otherwise}\end{cases}$$

$$s_1^p(t)=A\sqrt{T}\psi_1^p(t)\ ,\ s_2^p(t)=A\sqrt{T}\psi_2^p(t)\ ,\ s_3^p(t)=A\sqrt{T}\psi_3^p(t)\ ,\ s_4^p(t)=A\sqrt{T}\psi_4^p(t)$$

$$\mathbf{s}_1^p=\left[s_{11}^p,\ s_{12}^p,\ s_{13}^p,\ s_{14}^p\right]=\left[A\sqrt{T},\ 0,\ 0,\ 0\right]\ ,\ \mathbf{s}_2^p=\left[s_{21}^p,\ s_{22}^p,\ s_{23}^p,\ s_{24}^p\right]=\left[0,\ A\sqrt{T},\ 0,\ 0\right]$$

$$\mathbf{s}_3^p=\left[s_{31}^p,\ s_{32}^p,\ s_{33}^p,\ s_{34}^p\right]=\left[0,\ 0,\ A\sqrt{T},\ 0\right]\ ,\ \mathbf{s}_4^p=\left[s_{41}^p,\ s_{42}^p,\ s_{43}^p,\ s_{44}^p\right]=\left[\ 0,\ 0,\ 0,\ A\sqrt{T}\right]\quad (2.1)$$

It is clear from Fig. 2.1 and the expressions in (2.1) that as we go to higher dimensions, we slice the time axis more, this in turn increases our bandwidth requirement, So we can establish the following relation

$$\text{Number of dimensions } N\uparrow\quad \text{Bandwidth}\uparrow\qquad (2.2)$$

Asa result of this bandwidth increase, we should gain an increase in the minimum distance of the adjacent signal vectors.

By taking (2.1), we find as follows

$$d_{\min 4PPM}=d_{12p}=\left|\mathbf{s}_1^p-\mathbf{s}_2^p\right|=\left[\left(A\sqrt{T}-0\right)^2+\left(0-A\sqrt{T}\right)^2+\left(0-0\right)^2+\left(0-0\right)^2\right]^{0.5}=A\sqrt{2T}$$

$$d_{\min 4PSK}=d_{\min 4PPM}\qquad (2.3)$$

Comparing, (1.1) and (2.1), we see that the increase in the minimum distance of 4 PPM has not been achieved. The reason that is the signal vectors of 4 PPM are not distributed (in a optimum manner) to the far vertices of the 4 dimensional cube. This can be better understood by examining Fig. 1.2 of ECE 587_Notes on Codes (available at ece587.cankaya.edu.tr). The same figure is reproduced here for convenience as Fig. 2.2.

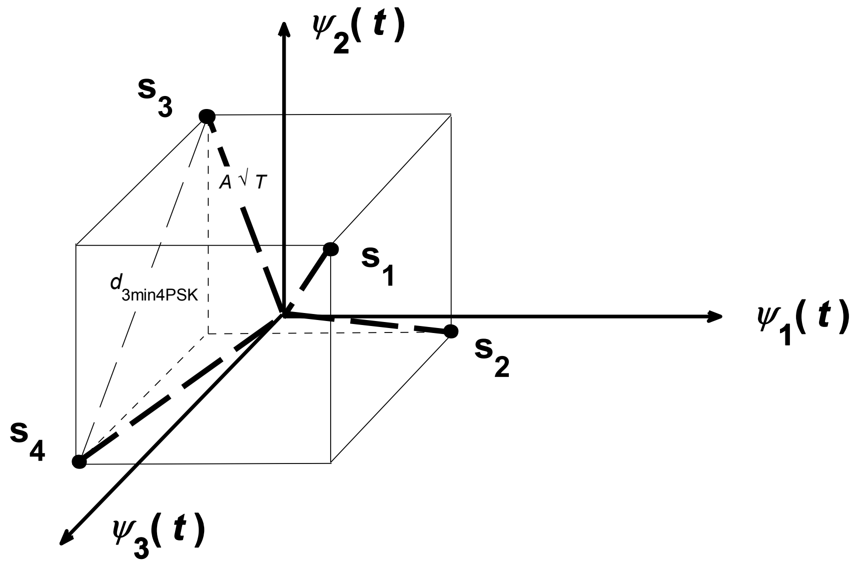

Fig. 2.2 The placement of 4 PSK signal in 3 dimensional constellation.

In a similar manner, we can modify the components of the signal vector in (2.1) and write (by using the same basis functions of (2.1))

$$\mathbf{s}_1^b = \left[s_{11}^b,\ s_{12}^b,\ s_{13}^b,\ s_{14}^b\right] = \left[A\sqrt{T/2},\ A\sqrt{T/2},\ 0,\ 0\right] \quad , \quad \varepsilon_1^b = A^2 T$$
$$\mathbf{s}_2^b = \left[s_{21}^b,\ s_{22}^b,\ s_{23}^b,\ s_{24}^b\right] = \left[0,\ A\sqrt{T/2},\ A\sqrt{T/2},\ 0\right] \quad , \quad \varepsilon_2^b = A^2 T$$
$$\mathbf{s}_3^b = \left[s_{31}^b,\ s_{32}^b,\ s_{33}^b,\ s_{34}^b\right] = \left[0,\ A\sqrt{T/2},\ 0,\ A\sqrt{T/2}\right] \quad , \quad \varepsilon_3^b = A^2 T$$
$$\mathbf{s}_4^b = \left[s_{41}^b,\ s_{42}^b,\ s_{43}^b,\ s_{44}^b\right] = \left[A\sqrt{T/2},\ 0,\ A\sqrt{T/2},\ 0\right] \quad , \quad \varepsilon_4^b = A^2 T \qquad (2.4)$$

Carrying out the minimum distance evaluations for (2.2), we get

$$d_{\min 4b} = d_{12} = \left|\mathbf{s}_1^b - \mathbf{s}_2^b\right| = \left[\left(A\sqrt{T/2} - 0\right)^2 + \left(A\sqrt{T/2} - A\sqrt{T/2}\right)^2 + \left(0 - A\sqrt{T/2}\right)^2 + \left(0-0\right)^2\right]^{0.5} = 2A\sqrt{T}$$

$$d_{\min 4b} > d_{\min 4PSK} = d_{\min 4PPM} \quad , \quad \frac{d_{\min 4b}}{d_{\min 4PSK}} = \sqrt{2} \qquad (2.5)$$

As seen from (2.5), at the expense of increasing the bandwidth requirement twice with respect to the 2 dimensional 4 PSK, we have acheived an minimum distance increase of $\sqrt{2}$.

Exercise 2.1 : Find and plot the time waveforms of $s_1^b\left(t\right)\cdots s_4^b\left(t\right)$.